

First Semester M.Tech. Degree Examination, Dec.2015/Jan.2016
Advanced Mathematics

Time: 3 hrs.

Max. Marks: 100

Note: Answer any FIVE full questions.

- 1 a. Construct QR factorization for the matrix :

$$A = \begin{bmatrix} -4 & 2 & 2 \\ 3 & -3 & 3 \\ 6 & 6 & 0 \end{bmatrix}$$

(10 Marks)

- b. Solve the system of equations :

$$x_3 + 2x_4 = 1$$

$$x_1 + 2x_2 + 2x_3 + 3x_4 = 2$$

in the least – square sense.

(10 Marks)

- 2 a. Apply the shifted QR algorithm to the matrix $A = \begin{bmatrix} 3 & 1 \\ 1 & 5 \end{bmatrix}$. Carry out three iterations.

(10 Marks)

b. Find the singular – value decomposition of the matrix $A = \begin{bmatrix} -3 & 1 \\ -2 & 1 \\ -1 & 1 \\ 0 & 1 \\ 1 & 1 \\ 2 & 1 \\ 3 & 1 \end{bmatrix}$.

(10 Marks)

- 3 a. Derive Euler – Lagrange's formula in the form $\frac{\partial f}{\partial y} - \frac{d}{dx} \left(\frac{\partial f}{\partial y'} \right) = 0$.

(06 Marks)

- b. Find the extremum of functional : $v \int_0^1 \frac{\sqrt{1+(y')^2}}{x} dx$, given $y(0) = 0$ and $y(1) = 1$.

(07 Marks)

- c. Solve the variation problem $v \int_1^2 [x^2(y')^2 + 2y(x+y)] dx = 0$, given that $y(1) = y(2) = 0$.

(07 Marks)

- 4 a. Find the function $y(x)$ for which $\int_0^\pi [(y')^2 - y^2] dx$ is stationary. Given that $\int_0^\pi y dx = 1$ and $y(0) = 0, y(\pi) = 1$.

(10 Marks)

- b. Find the extremals of the functional : $I = \int_0^{\pi/2} [(y')^2 + (z')^2 + 2yz] dx$, given that $y(0) = 0$,

$$y\left(\frac{\pi}{2}\right) = 1, z(0) = 0, z\left(\frac{\pi}{2}\right) = -1.$$

(10 Marks)